

# Erratum to “Generalizations of the Poincaré-Birkhoff theorem”

By JOHN FRANKS\*

Theorem (2.1) of [1] is not stated correctly. The correct statement (and the result which is actually proved) is the following.

**THEOREM.** *Suppose  $f : \mathbb{A} \rightarrow \mathbb{A}$  is an orientation preserving homeomorphism of the open annulus which is homotopic to the identity, and satisfies the following conditions:*

1. *Every point of  $\mathbb{A}$  is non-wandering.*
2.  *$f$  has at most finitely many fixed points.*
3. *There is a lift of  $f$  to its universal covering space,  $\tilde{f} : \tilde{\mathbb{A}} \rightarrow \tilde{\mathbb{A}}$ , which possesses both a positively returning disk, which is a lift of a disk in  $\mathbb{A}$ , and a negatively returning disk, which is a lift of a disk in  $\mathbb{A}$ .*

*Then  $f$  has a fixed point of positive index.*

Other results of [1] which rely on Theorem (2.1) are correct as stated and follow from the result above.

A disk  $U$  in the covering space  $\tilde{\mathbb{A}}$  is called *positively returning* for  $\tilde{f}$  if  $\tilde{f}(U) \cap U = \emptyset$  and  $\tilde{f}^n(U) \cap T^k(U) \neq \emptyset$  for some  $n, k > 0$ , where  $T$  is the generator of the infinite cyclic group of covering translations. Negatively returning disks are defined similarly. The difference in the statement of the theorem above and the statement of Theorem (2.1) of [1] is the additional requirement in item 3 that the positively and negatively returning disks in the covering space are lifts of disks in the annulus. Equivalently the positively and negatively returning disks in the covering space must be disjoint from their image under  $T^k$ ,  $k \neq 0$  (standard arguments show it suffices that  $T(U) \cap U = \emptyset$ ). This additional hypothesis is satisfied in other results from [1] which use Theorem (2.1).

I do not know whether Theorem (2.1) of [1], as originally stated, is true or not.

---

\*Supported in part by NSF grant DMS-0099640.

NORTHWESTERN UNIVERSITY, EVANSTON, IL  
*E-mail address:* john@math.northwestern.edu

## REFERENCES

- [1] J. FRANKS, Generalizations of the Poincaré-Birkhoff theorem, *Ann. of Math.* **128** (1988), 139–151.

(Received October 13, 2004)